How to test for bubbles: discount rates and volatility

Background

- We want to test
$$\{H_0 : B_t = 0\} \text{ vs. } \{H_1 : B_t > 0\}$$
- in
$$P_t = \underbrace{\mathbb{E}_t\left[\sum_{i=1}^{\infty} \frac{D_{t+i}}{\prod_{j=0}^{i-1} R^e_{t+j}}\right]}_{=F_t,\ \text{fundamental value}} + B_t.$$
R^e_t discount rate, D_t dividend.
- Since F_t is hard to compute, the typical approach is to test **auxiliary hypotheses**.

Objective

Evaluate size and power of tests of auxiliary hypotheses when applied to an estimated model of fundamental value.

Auxiliary test 1: explosive growth

- B_t fulfills
$$R^e_t B_t = \mathbb{E}_t[B_{t+1}].$$
- Test $H_0^{\text{ex}} : a_t = 1$ vs. $H_1^{\text{ex}} : a_t > 1$, where a_t price growth rate.
- Implementation: right-sided Augmented Dickey Fuller (ADF) regression
  - on full sample (ADF, Diba and Grossman, 1988).
  - recursively on subsamples (SADF, Phillips et al., 2011).
- Shortcoming: fundamental price may grow explosively.

Auxiliary test 2: cointegration (CI)

- Assuming $R^e_t = R$ for all t, we have (Campbell and Shiller, 1987)
$$F_t - \frac{R^{-1}}{1-R^{-1}} D_t = \frac{1}{1-R^{-1}} \sum_{i=1}^{\infty} \mathbb{E}_t\left[R^{-i}\Delta D_{t+i}\right].$$
- Test H_0^{ci}: no cointegration between prices and dividends. Corresponds to H_1.
- Implementation: Engle-Granger two-step regression.
- Shortcoming: constant discount rate assumption.

Auxiliary test 3: variance decomposition (VD)

- An approximation to F_t (Cochrane, 1992, 2011) yields
$$Var(PD_t) = \frac{1}{1-\Omega}\sum_{j=1}^{\infty} \Omega^j Cov(PD_t, \Delta d_{t+j}) - \frac{1}{1-\Omega}\sum_{j=1}^{\infty} \Omega^j Cov(PD_t, r_{t+j}).$$
- PD_t price-dividend ratio under H_0, $\Omega = e^{\mathbb{E}[\Delta d_t] - \mathbb{E}[r_t]}$.
- H_0^{vd}: LHS = RHS vs. H_1^{vd}: LHS > RHS.
- Implementation: Vector Autoregression in $(r_t, \Delta d_t, pd_t)$.
- Shortcoming: long-horizon predictions are hard.

Present value model

- Transitions for monthly dividend growth and log discount rate:
$$\Delta d_{t+1} = \mu_d + \sigma_d \sqrt{h_{t+1}} \epsilon_{d,t+1}$$
$$r^e_{t+1} = \mu_r + \varphi_r (r^e_t - \mu_r) + \sigma_r \sqrt{h_{t+1}} \epsilon_{r,t+1}.$$
- Variance process:
$$h_{t+1} = \alpha \epsilon^2_{v,t} + \beta h_t.$$
- Proxy of variance process:
$$RV_t = \gamma + h_t + \sigma_{rv} \epsilon_{rv,t}.$$
- $(\epsilon_{d,t+1}, \epsilon_{r,t+1}, \epsilon_{v,t+1})'$ standard normal.
- Using the approach of Ang and Liu (2004), we get, under H_0:
$$PD_t = \sum_{i=1}^{\infty} \exp(a_i + b_i r^e_t + c_i h_{t+1}),$$
with a_i, b_i and c_i given recursively.

References: see the paper

Estimation

- Data: value-weighted CRSP files, 1927 to 2016. RV_t computed from daily returns.
- Simulated Score Method (Gallant and Tauchen, 1996).
- Two stages:
  1. Auxiliary model for monthly realized variance:
  $$g_t = \eta_0 RV_{t-1} + \eta_1 g_{t-1}$$
  $$RV_t = \eta_2 + g_t + \eta_3 v_t; \quad v_t \sim \mathcal{N}(0,1).$$
  $(\alpha, \beta, \gamma, \sigma_{rv})$ set to minimize a quadratic form of the realized variance model scores.
  2. Yearly return forecasting VAR:
  $$\begin{pmatrix} r^a_t \\ pd^a_t \end{pmatrix} = \begin{pmatrix} \eta_5 \\ \eta_6 \end{pmatrix} + \begin{pmatrix} \eta_7 & \eta_8 \\ \eta_9 & \eta_{10} \end{pmatrix} \begin{pmatrix} r^a_{t-12} \\ pd^a_{t-12} \end{pmatrix} + \begin{pmatrix} u_{1,t} \\ u_{2,t} \end{pmatrix}, \quad \begin{pmatrix} u_{1,t} \\ u_{2,t} \end{pmatrix} \sim \mathcal{N}\left(0, \begin{pmatrix} \eta_{11} & \eta_{12} \\ \eta_{12} & \eta_{13} \end{pmatrix}\right), \quad t \in \{12, 24, \ldots\}.$$
  The remaining parameters of the DGP are set to minimize a quadratic form of the VAR scores.
- Estimates:

	$\hat\mu_d$	$\hat\mu_r$	$\hat\varphi_d$	$\hat\sigma_d$	$\hat\sigma_r$	$\hat\alpha$	$\hat\beta$	$\hat\gamma$	$\hat\sigma_{rv}$
est	0.0021	0.0079	0.9965	0.2146	0.0034	0.0021	0.7597	-0.0061	0.0024
s.e.	0.0007	0.0029	0.0025	0.4834	0.0077	0.0004	0.0741	0.0022	0.0008

$Q_n(\hat\theta_2) = 8.668$, p-value: 0.07

Results: conventional critical values are faulty...

- Simulate under H_0 and run tests.
- Rejection frequencies of tests for explosive growth at five percent level:

$T =$	100	250	500	1000
ADF	0.0814	0.1238	0.1761	0.2787
SADF	0.1873	0.3493	0.5711	0.8481

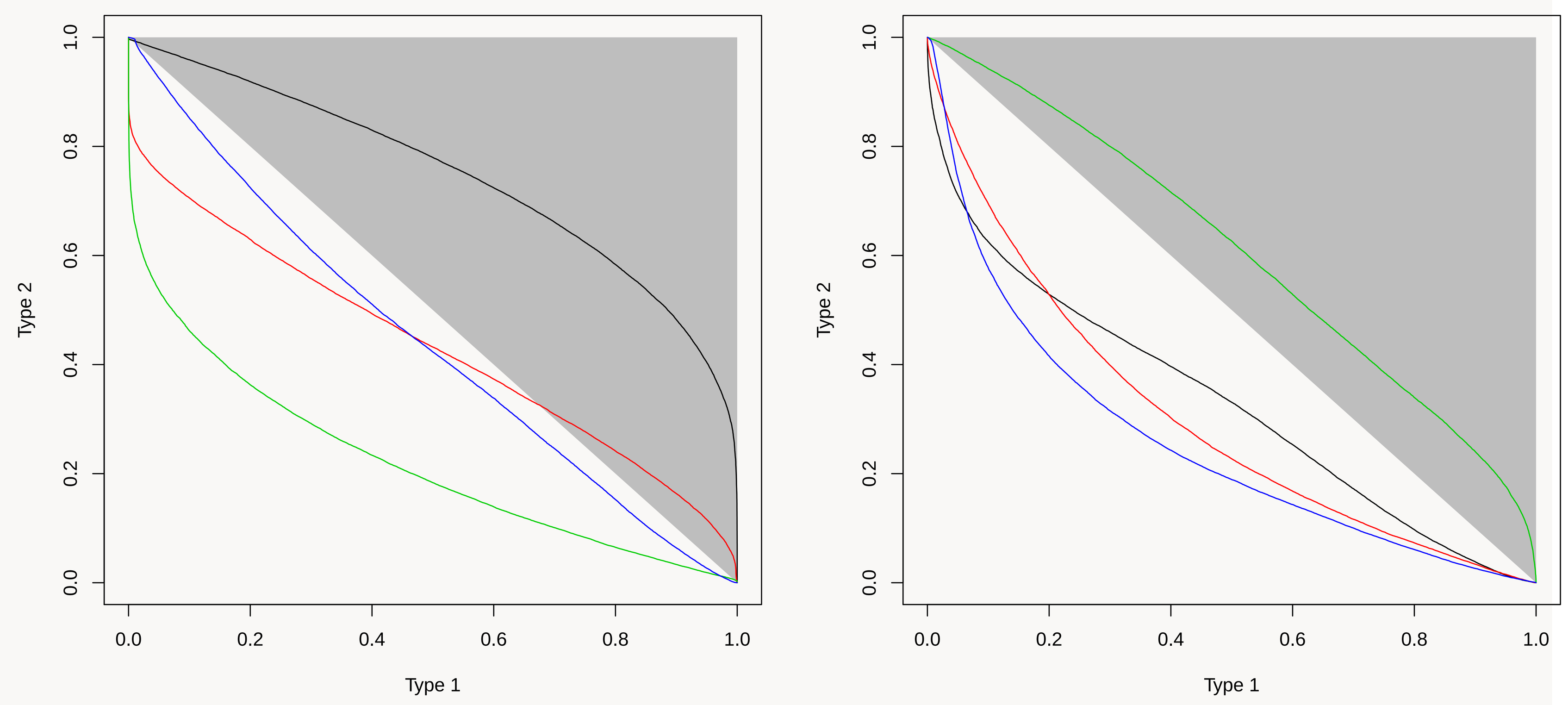
Desired value of a cell: 0.0500.
- Rejection frequencies of cointegration test at five percent level:

$T =$	100	250	500	1000
CI	0.5919	0.6056	0.6387	0.7167

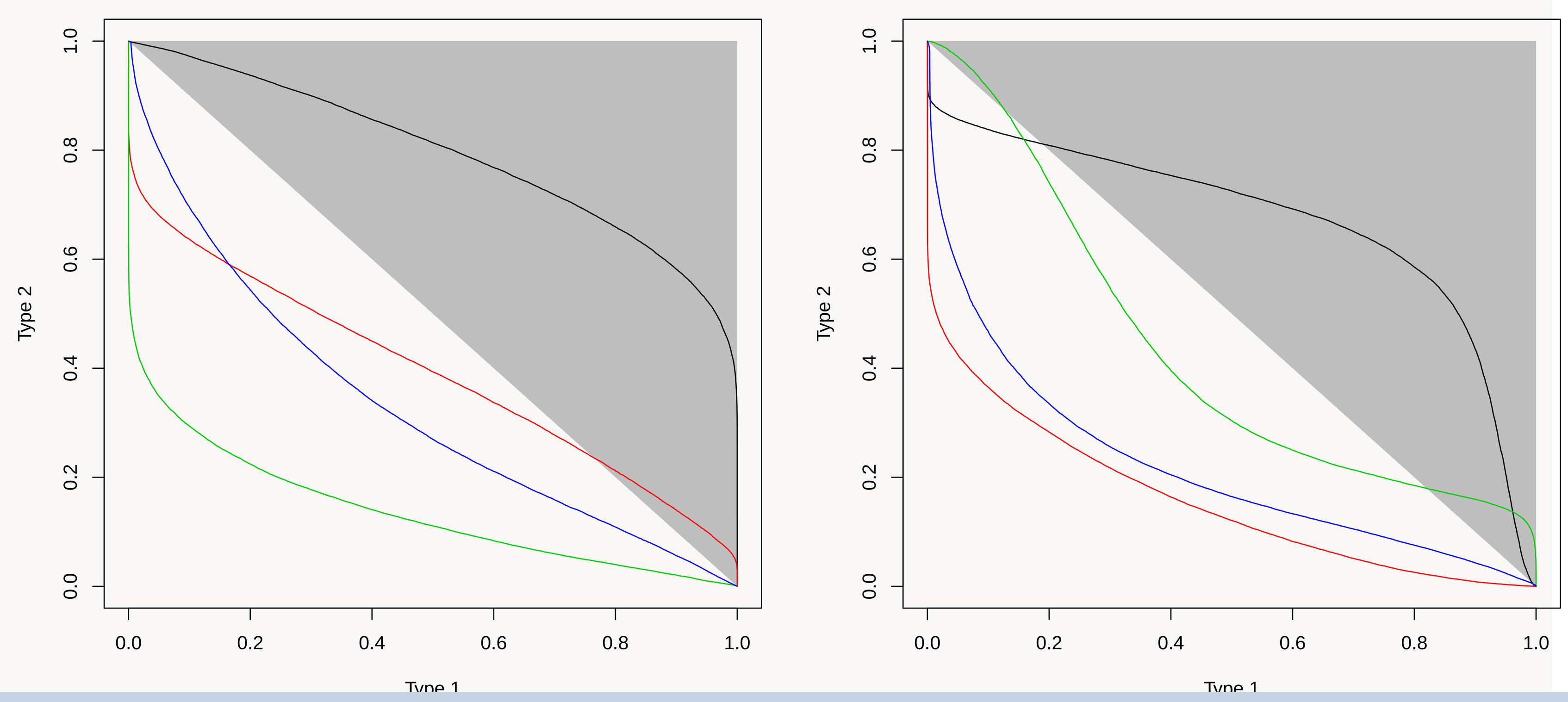
Desired value of a cell: 1.0000.

... but auxiliary tests are still useful.

- Compare test statistics on data generated under H_0 and H_1.
- Bubble process H_1:
$$B_{t+1} = \theta_{t+1}\pi^{-1}\left(R^e_t B_t - \underline{B}\right) + \underline{B},$$
with θ_{t+1} Bernoulli process. Variation of Blanchard and Watson (1982).
- Draw Lines of Enlightened Judgement (Leamer, 1978).
Legend: Left: $\pi = 0.5$, right: $\pi = 0.99$. $\underline{B} = 1$. Colors: ADF, SADF, CI, VD.
Grey area: random or perverse judgement.
  - $T = 100$:



  - $T = 500$:



Matthias Schmidtblaicher
matthias.schmidtblaicher@eui.eu
European University Institute